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# A Fast and Fair Algorithm for Distributed Subcarrier Allocation Using Coalitions and the Nash Bargaining Solution

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**Abstract**—In this paper we present a distributed, fast and fair resource allocation algorithm for a multiuser, wireless LTE OFDMA channel. Extending our previous work, we propose additional efficiency enhancements and discuss the protocol issues involved. We further explore the trade-offs between performance and costs and present an analysis of the overhead and time requirements. The proposed algorithm partitions the users of the wireless network into coalitions and, using the game theoretic concept of the Nash Bargaining Solution (NBS), offers a cooperative solution to the subcarrier allocation problem. The use of the NBS ensures that the fairness provided matches that offered by the widely accepted Proportional Fair (PF) scheduler. Simulation results show that the sum rate achieved can be tuned to be almost equivalent to the sum rate of the PF scheduler, while only requiring limited resources and information exchange. At the same time, the extensive efficiency enhancements and its distributed nature render the algorithm fast and low-complexity enough to be suitable for use in a real-time wireless system.

## I. INTRODUCTION

In this paper we propose a distributed resource allocation algorithm for a multiuser, frequency selective wireless channel, where the existence of independent communication links between the users and the base station (BS) and their instantaneous fluctuation allows for the benefits of multiuser diversity to be harvested. The majority of the existing work assumes that a master device takes responsibility for dictating the resource allocation process. This simplifies protocol issues, but the various costs associated with this approach cannot be ignored; the master device needs information about the channel quality for all devices for which it is responsible. Furthermore, the allocation decisions have to be sent back to wireless devices, thus altogether generating an overhead that impedes system performance.

The aforementioned costs suggest that there are potential benefits when the resource allocation process takes place in a more distributed fashion. Coalition formation [1] is a game theoretic concept that can be used to achieve this and it can be used in a central or self-organized version. Expensive, in terms of complexity and time required, algorithms exist for optimal solutions for the former; the latter is robust, scales

well, and uses simple heuristics to form beneficial coalitions at low complexity.

In this paper we extend the work presented in [2], where a fast and fair distributed algorithm for subcarrier allocation was proposed. Here, we propose additional efficiency enhancements, discuss the protocol issues involved and investigate the algorithm's performance in relation to the size of the coalitions formed. We discuss how the algorithm scales with the number of users, we further explore the trade-offs between performance and costs and, finally, we present an analysis of the overhead and time requirements. A key benefit of our work is the ability to select parameter values that significantly reduce time and overhead requirements, while retaining a very large part of the maximum achievable sum rate.

## II. RELATED WORK

A thorough presentation of game theory application in wireless channel resource allocation can be found in [3]. Authors in [4] argue that cooperative games are not well suited to the distributed nature of wireless networking, as additional signaling between decision makers is required. However, results presented in this paper contradict this, as signaling and overhead requirements are limited when compared to the centralized PF scheduler. NBS [5] and coalitions are used in [6] to achieve fair resource allocation for multiuser, OFDMA wireless networks. A partially distributed scheme is proposed, where the Base Station simply acts like a market place where the bargaining of subcarriers among the users takes place. A similar scheme, but with reduced complexity, is proposed in [7]. Finally, distributed approaches for fair resource allocation in wireless networks using NBS are presented in [8] and [9].

The novelty of our work is the key focus on low complexity. Most of the existing literature discusses the wireless resource allocation problem mainly in terms of optimal power, rate and fairness performance and largely neglects complexity issues. Our work proposes a low-complexity, fast and fair distributed algorithm with limited signaling and overhead requirements, suitable for implementation in a real-time system.

### III. SYSTEM MODEL AND DESCRIPTION

We focus on the downlink, single antenna (for all devices) scenario of the SCM LTE channel presented in [10]. The network consists of a single BS with users randomly scattered around it; Table I presents the channel and system parameters that we used in our simulations in more detail. The channel quality each individual user experiences is strongly dependent on its distance from the BS and therefore users' SNR values vary accordingly. All 1024 channel subcarriers are used and no subcarrier sharing is allowed. We assume that there exists a reliable and fast feedback channel for users to exchange the information required during the allocation process; this is also used for the allocation decisions to be fed back from the mobile devices to the BS. Our performance metric is the rate achieved by each user; we aim to maximize the sum rate of all users, while maintaining a fair operation point for all. We calculate the theoretical rate of user  $k$  in subcarrier  $s$  by using (1), where  $W$  is the channel's bandwidth,  $S$  is the number of subcarriers in the channel and  $H_{k,s}$  is the channel gain for user  $k$  in subcarrier  $s$ .

$$R_{k,s} = \frac{W}{S} \times \log_2 (1 + SNR \times \|H_{k,s}\|^2) \quad (1)$$

Finally, the sum rate (2) achieved by user  $k$  for the whole channel simulation is calculated by adding up the rates for all the subcarriers the user was allocated.  $S_k$  is the vector of subcarriers allocated to user  $k$  and  $y$  is the number of simulated channel realizations (i.e. 2000).

$$R_k = \sum_{i=1}^y \sum_{S_k} R_{k,s} \quad (2)$$

### IV. NASH BARGAINING SOLUTION

As detailed in [6], the *game* is the subcarrier allocation problem, the *players* (i.e. intelligent and rational decision-making entities participating in the game) are the devices in the wireless network and the *goal* is to maximize the chosen utility function for all players simultaneously. Similarly to [6] and to the vast majority of the relevant literature, we choose the data rate achieved by the device-user (these terms are used interchangeably throughout the paper) in a single channel realization to serve as the utility function (3):

$$U = \sum_{S_k} R_{k,s} \quad (3)$$

In order to determine the NBS, we need to find the subcarrier allocation matrix that maximizes the product of data rates (i.e. the Nash function) for the users-members of each coalition:

$$\prod_{k=1}^m (R_k - R_{min}^k) \quad (4)$$

In (4),  $m$  is the coalition size,  $R_k$  the sum rate achieved by user  $k$  over the allocated subcarrier groups (i.e. subchannels) and  $R_{min}^k$  the minimal rate requirement of user  $k$ . However, depending on each user's choice of  $R_{min}^k$ , (4) might not converge to a solution. To avoid this complication we set  $R_{min}^k$  equal to zero for every user; this also improves the execution

TABLE I  
SYSTEM & CHANNEL SIMULATION PARAMETERS

Carrier Frequency	2 GHz
Transmission Bandwidth	10 MHz
Time Slot / Sub-frame duration	0.5 ms
Number of subcarriers	1024
Base Station transmit power	43 dBm (20W)
Propagation Model	SCM Urban Macro
Noise Power	-104 dBm
Cell radius	150 m
MS velocity	10 m/s

speed of the algorithm and provides [6] a proportionally fair behavior to the allocation process.

As described in [6], the unique NBS (i.e. the subcarrier allocation matrix in our system) that maximizes (4) satisfies the following axioms: (i) *Individual Rationality*, (ii) *Feasibility*, (iii) *Pareto Optimality*, (iv) *Independence of Irrelevant Alternatives*, (v) *Independence of Linear Transformations* and (vi) *Symmetry*. These axioms ensure that the NBS maximizes all  $R_i$  simultaneously and also provides a fair operating point for all participating players.

### V. THE ALLOCATION ALGORITHM

#### A. Coalition Formation

We keep algorithm complexity low by avoiding the search for the optimal partitioning of users into coalitions. Instead, and similar to [6], we choose to form equally-sized coalitions. The choice of non-predefined coalition sizes would provide better exploitation of multiuser diversity and higher sum rates, but would also make the allocation algorithm significantly slower. Our approach is straightforward; all possible partitions of the users into equally-sized coalitions are generated and, after bargaining for subcarriers within each coalition has finished, the appropriate coalition structure (i.e. partitioning of users into coalitions) has to be selected. The number of coalitions formed is:

$$c = \begin{cases} N/m & , \text{ if } N \equiv 0 \pmod{m} \\ \lceil N/m \rceil + 1 & , \text{ otherwise} \end{cases} \quad (5)$$

$N$  is the number of users in the network and  $m$  is the coalition size. If the users cannot be exactly split into equally-sized coalitions, the remaining users form a single, smaller-sized coalition. In the case of a single remaining user, this user is part of no coalition and simply gets the remaining unallocated subcarriers. The allocation of equal number of subcarriers to every user (see next section) guarantees fairness to all users.

#### B. Bargaining for Subcarriers

After forming all possible coalition structures, the utility that each structure can yield has to be calculated through bargaining within each coalition. The array of all possible subcarrier-user permutations is generated and all permutations are examined to determine which one generates the highest utility (i.e. product of rates for all coalition members). Since these computations take place within each coalition, limited

signaling is required between coalition members in order to exchange the information (i.e. achievable rates for each subcarrier) necessary. However, the very large number of permutations makes the exhaustive search for the optimal permutation simply infeasible and, therefore, we choose to make some major modifications to this approach:

1) *Subcarrier Grouping*: Allocating subcarriers one by one is a complex process, requiring the testing of  $m^t$  subcarrier permutations, for  $m$  users and  $t$  subcarriers per coalition. Our approach, similar to that of many practical systems, is to group adjacent subcarriers into groups and allocate them as a single unit. This incurs a slight loss of sum rate performance, as a degree of fine-grained control over the allocation process is lost. Generally, forming subcarrier groups of size  $s$  provides a  $m^{t \times \frac{(s-1)}{s}}$  - fold increase in speed. An additional benefit is the similarly reduced memory requirements, something very crucial in devices with limited resources.

2) *Equal number of subcarriers*: As discussed in [2], the allocation of the same number of subcarrier groups to every user maintains fairness almost identical to proportional fairness. This strategy hugely reduces the number of subcarrier permutations that need to be tested and makes the algorithm significantly faster, while only slightly (i.e. up to 5% according to simulation results) impacting achieved rates.

The number of permutations for each coalition after these reductions is shown in Table II; this number is very big for the smaller values of  $N$ , as in that case fewer coalitions are formed and each one gets a larger number of subcarrier groups, hence the increased number of permutations.

### C. Selection of Coalition structure

The last stage of the allocation process is the selection of the ‘winning’ coalition structure. Our simulations indicate that almost identical fairness and sum rates are achieved across all structures and that the choice of a specific structure only marginally (i.e. up to 1% in terms of sum rate) changes the outcome of the allocation process. This behavior stems from the choice of equal number of subcarriers per user and from setting  $R_{min}^k$  equal to zero for every user. It also offers the opportunity to further enhance the algorithm’s efficiency by reducing the number of coalition structures tested.

### D. Efficiency Enhancements

1) *Permutations sampling*: Based on the observation that no significant differences from permutation to permutation exist, we further reduce the number of permutations that need to be tested by only testing a sample of them. Sampling at a ‘rate’ of  $1/p$  increases the algorithm execution speed by a factor of  $p$  and only marginally (e.g. 4% for  $N=10$  and  $p=10^3$ ) reduces sum rate, as shown by our simulations.

2) *Realizations step*: Repeating the allocation process less often than every channel realization yields a great increase in algorithm efficiency and only marginally reduces sum rate. Repetition of the process every  $r$  realizations provides an  $r$ -fold increase in algorithm speed. In [2] we showed that setting  $l$  equal to 10 only induces a 4% loss in sum rate.

TABLE II  
NUMBER OF COALITION STRUCTURES AND PERMUTATIONS

Number of structures	N	m	Structures	Permutations
$st = \prod_{i=0,1,\dots}^{N-i \times m} \binom{N-i \times m}{m} = \frac{N!}{(\frac{N}{m})!}$	8	2	105	$\approx 2 \times 10^6$
	8	4	70	369600
	10	2	945	184756
	10	5	252	113400
(N: number of users)	12	2	10395	48620
(m: coalition size)	14	2	135135	3433

3) *Number of coalition structures*: Since the effect of selecting a specific coalition structure is marginal, we reduce the number of coalition structures tested. As shown in Table II, when  $N$  increases to values larger than 10,  $st$  becomes very large; sampling at a ‘rate’ of  $1/q$  increases execution speed by a factor of  $q$ . Simulation results show that, even for large (e.g.  $\geq 0.9$ ) values of  $q/st$ , the loss in sum rate rarely exceeds 2%. As a result, the application of the proposed algorithm can be extended to networks with large number of users by allowing  $q/st$  to take large values.

### E. Protocol analysis

The distributed nature of the algorithm stems from the fact that allocation decisions are made at a local level, as each coalition decides with subcarrier-user permutation will be used. The utility computation requires that each coalition member transmits its achievable data rate (for every subcarrier group) to the ‘master’ user of the coalition. After performing the necessary calculations and determining which permutation provides the highest utility, the master user transmits the result (i.e. total data rate for all members) to the ‘leader’ user of the network, who in turn makes the selection of the appropriate coalition structure (the leader remains the same throughout the whole allocation process) and transmits the allocation decision back to the BS. It is also the leader’s responsibility to randomly pre-assign an equal number of subcarrier groups to each coalition at the start of the process. The aforementioned signaling process is repeated as often as the *realizations step* dictates. The roles of ‘master’ and ‘leader’ are randomly assigned to ‘willing’ users, who declare their ‘willingness’ by appropriate beaconing at the start of the allocation process. There must be one master user within each coalition and one leader user in the network. A master user can also be the leader. If no users are willing to assume these roles (e.g. due to low battery level) they are randomly assigned, with a preference towards more powerful users (e.g. laptop preferred over smartphone). Additionally, the existence of willing users in each coalition can favor the selection of a specific coalition structure over another one that does not have a willing user available for every coalition. In future work, it would be worthwhile exploring the possibility of ‘rewarding’ willing users to increase their incentive to assume these roles; this could come in the form of allocating extra subcarrier groups to these users, as long as the overall fairness among users does not suffer. Periodic beaconing ensures that the algorithm adapts as users join and leave the network. This generates only

Fig. 1. Sum rate comparison for different coalition sizes

minimum overhead, as the repetition of the allocation process is significantly more frequent than that of the beaconing one since the time-scale of changes in the network (i.e. users leaving/joining) is much slower than instantaneous fluctuation in signal quality. As a failsafe strategy, when a master or the leader user leaves the network, the BS continues with the previous allocation until beaconing takes place again and the algorithm adapts to the new network form.

## VI. RESULTS

We compare the proposed algorithm against the PF scheduler [11], due to its wide acceptance both in literature, as well as in actual products. Additionally, many of the game theoretic schedulers proposed in the literature are not implementable in practice due to their complexity. The time window used for the PF scheduler is 500-subcarriers long; simulations showed that this provides a good balance between sum rate and short-term fairness. We implemented a standard PF [11] scheduler in JAVA<sup>TM</sup> that allocates resources on a per-subcarrier basis; this provides higher sum rates at the expense of increased computational complexity. Our NBS scheduler is also implemented in JAVA<sup>TM</sup>, while the LTE SCME channel model was simulated using Matlab<sup>®</sup>. Simulations were performed for 2000 different channels (i.e. different users' locations), with each channel being simulated for 2000 realizations (i.e. uncorrelated instances of small scale fading effects); results presented in this section are averaged over the 2000 different channels. We use 10-sized subcarrier groups, as our simulations showed that this offers an excellent trade-off between sum rate performance and algorithm efficiency, for the specific channel model simulated; a different value might be optimal for channels with higher or lower coherence bandwidth. We examine the algorithm's behavior and performance with up to 10 users; it is impractical, in terms of time and memory required for full 1024-subcarrier 2000 realization channel simulation, to evaluate the full complexity algorithm for larger number of users. This limitation only applies to channel simulation; the efficiency enhancements to the allocation algorithm and its distributed nature allow it to scale well with large number of users.

### A. Sum rate & Fairness

We use the sum rate achieved by the users over the duration of the simulation (i.e. 2000 realizations or 2 seconds) as the

Fig. 2. Effect of 'Realization' and 'Structure' step

basic performance metric. The proposed algorithm yields a sum rate that can be tuned (i.e. by choice of parameter values: *coalition size*, *permutation*, *realization* and *structure step*) to be equivalent to up to 90% of the PF sum rate on average between different simulation sets. This averaged value varies between 69% and 108% of the PF sum rate, as the users' long-term channel qualities vary significantly between the different channels that are simulated, due to the random location of the users around the BS. This is an excellent result, given the speed improvement over the PF scheduler.

To investigate the fairness achieved by the proposed algorithm we calculate Jain's fairness index [12]:

$$Fairness = \frac{(\sum_{k=1}^N R_k)^2}{N \times \sum_{k=1}^N (R_k)^2} \quad (6)$$

$R_k$  represents the data rate achieved by user  $k$  over the whole simulation of the channel and  $N$  is the number of users in the channel. Results show that Jain's index achieved by the proposed algorithm (when the coalition structure with the lowest Jain's index value is selected) is consistently between 90% and 105% of Jain's index of the PF scheduler, with the average value being 99.5%. More insight into the fairness and sum rate performance of the algorithm is provided in our previous work in [2].

We also investigate the effect of coalition size. Simulation results show that sum rate increases as the size of the coalition increases, since a larger coalition is allocated more subcarriers and thus offers a wider range of subcarrier group permutations among coalition members; this allows for multiuser diversity benefits to be harvested in a more efficient way. This is illustrated in Fig. 1, for the case of three networks consisting of 6, 8 and 10 users respectively. It should be noted that increasing the coalition size to values larger than 5, even for networks with a large number of users, carries a significant penalty, as a much greater number of permutations needs to be tested. If *permutation step* is increased to counter this effect, the benefits of choosing a larger coalition size will be counterbalanced by the loss of a percentage of the sum rate due to the larger permutation step.

### B. Algorithm Efficiency & Overheads

A key benefit of our algorithm is the ability to select values for coalition size, permutation, realization and structure

TABLE III  
NUMBER OF OPERATIONS REQUIRED

Operation	NBS	PF
additions	$R \times ST \times m \times$ $P \times (N_{group}/c)$	$R \times S \times N \times$ $(ws + 1)$
mult./div.	$R \times ST \times m \times P$	$R \times S \times N$
comparisons	$R \times ST \times m$	$R \times S \times N$

Fig. 3. Time & overheads comparison

step that significantly reduce time and overhead requirements, while retaining a very large part of the achievable sum rate. A representative sample of this ability is presented in Fig. 2, for a network with 10 users; selecting higher values for the *realization step* only slightly affects sum rate, while time and overhead requirements are minimized. Additionally, the effect on sum rate of increasing *structure step* is negligible, but the time required for the algorithm to run is greatly reduced.

Similar gains can be observed when comparing the algorithm against the PF scheduler and against the same, but centralized, NBS algorithm. This is illustrated in Fig. 3, where normalized results for a network with  $N=10$  users,  $m=2$ , *structure step*=10 and *permutation step*=1000 are presented. ‘Time required (simulation)’ on the graph represents time measurements made during simulations and ‘Time required (prediction)’ has been calculated using Table III. Therein,  $P$  is the number of permutations tested,  $m$  the coalition size,  $ST$  the number of coalition structures tested,  $R$  the number of times the algorithm was repeated in a single simulation (i.e. 2000 realizations),  $N$  the total number of users,  $S$  the number of subcarriers,  $N_{group}$  the total number of subcarrier groups and  $ws$  the window size (measured in subcarriers) used for the PF scheduler. Overheads have been calculated using (7), considering the achievable data rate of a single user over a single subcarrier group as the ‘overhead unit’:

$$o = \begin{cases} P \times (m - 1) \times ST \times c \times R, & \text{for dist. NBS} \\ N \times R \times N_{group}, & \text{for cent. NBS} \\ N \times R \times S, & \text{for PF} \end{cases} \quad (7)$$

In (7) and Table III  $c$  is the number of coalitions formed. The difference between the distributed and centralized NBS is that in the distributed case the operations are off-loaded to the users (instead of the BS) and so the process is significantly sped up. In these calculations we choose to ignore the beaconing overheads, as beaconing takes place significantly less often than the allocation process and also requires much less information to be transmitted.

## VII. CONCLUSIONS & FUTURE WORK

A low-complexity, fast and fair distributed subcarrier allocation algorithm was presented in this paper. By utilizing the concept of Coalition Formation and Nash Bargaining, this algorithm achieves a sum rate that can be a close match (up to 90%, on average) of the sum rate offered by the widely

used PF scheduler. The achieved fairness almost replicates proportional fairness, providing a fair operating point for all users. The ability to tune algorithm parameters such as coalition size, *permutation*, *realization* and *structure step* offers the opportunity to minimize execution time and overheads, makes the proposed algorithm suitable for implementation in real-time systems and allows it to scale up well for networks with a large number of users.

In the future we plan to investigate in detail the effects of moving away from coalitions of pre-defined size; the gains and drawbacks of using dynamically-sized coalitions will be examined. Additionally, we plan to lift the restriction of zero minimal rate requirement for all users and introduce QoS provision to our allocation algorithm, so as to accommodate the requirements of next generation networks where heterogeneity and personalization are major aspects.

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